Beam Physics Note 63

25/04/02

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Luminosity Considerations for the LHC

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Keywords: luminosity, density functions

Summary

The usual expression is considered for the luminosity of an accelerator for bunched beams with a crossing angle and specific calculations are made for the LHC.

1 Introduction

Particles, in accelerators, travel in bunches or form a continuous beam. They either have head-on collisions at the interaction points or collide at a small crossing angle ϕ. In the case of the LHC, they travel in bunches and collide at a total angle of $\phi = 300\ \mu$rad. A bunch can be described by considering the number of particles it contains, N, and the normalised density, $\rho(x, y, s, s_0)$ [1]. Here s_0 refers to the centre of the bunch and the abbreviation $s = ct$ ($s_0 = ct_0$) has been used. In what follows it is assumed that there is no coupling between the three spatial coordinates and that the beams move in the s direction with x and y being the two transverse directions. The beams are also assumed to have Gaussian profiles in all three directions, namely

$$\rho^i(x, y, s, s_0) = \rho^i_x(x)\rho^i_y(y)\rho^i_s(s - s_0)$$

where $i = 1, 2$ (depending on the beam) and

$$\rho^i_z(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp -\frac{z^2}{2\sigma_z^2}, \qquad\qquad i = 1, 2,\ z = x, y,$$

$$\rho^i_s(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp -\frac{(s \pm s_0)^2}{2\sigma_s^2},$$

and the σ_k's ($k = x, y, s$) represent the standard deviation or beam sizes in the three coordinates.

The crossing angle ϕ is assumed to lie in the (x, s) plane and the beams can be represented by two rotations as follows

$$\begin{array}{ll} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = x\sin\frac{\phi}{2} + s\cos\frac{\phi}{2}, \\ x_2 = -x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_2 = x\sin\frac{\phi}{2} - s\cos\frac{\phi}{2} \end{array} \tag{1}$$

as shown in figure (1) below.

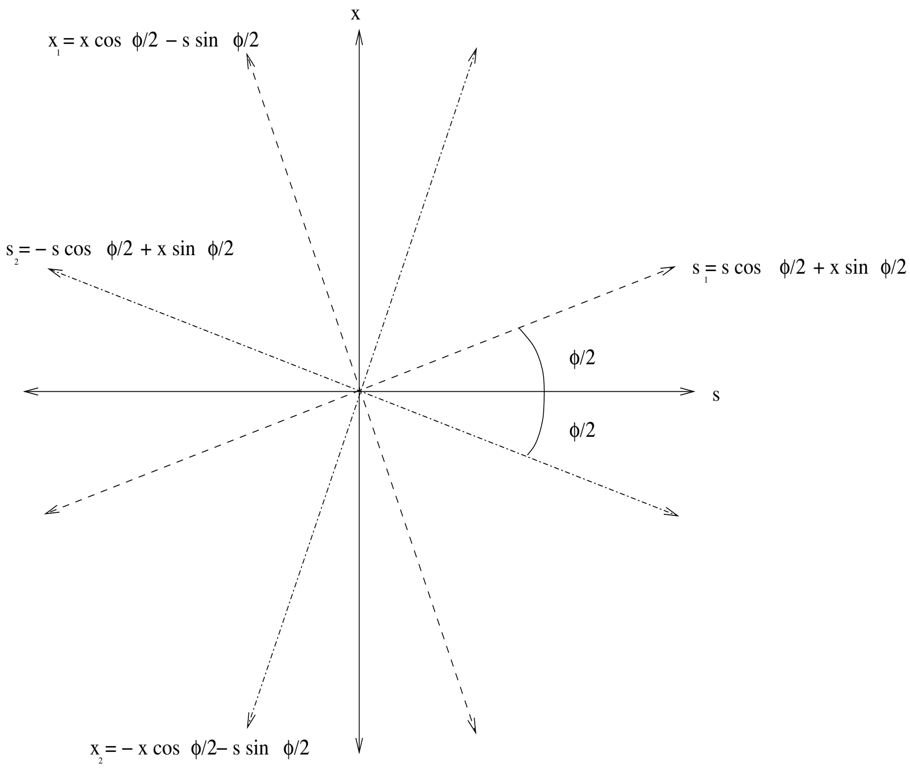

Figure 1: Geometry of crossing region

For a typical accelerator with bunched beams the expression for the luminosity is given by

$$\mathcal{L} = 2N_1N_2fB\cos^2\frac{\phi}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho_x^1(x)\rho_y^1(y)\rho_s^1(s-s_0)\rho_x^2(x)\rho_y^2(y)\rho_s^2(s+s_0)dxdydsds_0$$

where $s = ct$, f is the revolution frequency, B is the number of bunches per beam and ϕ is the crossing angle.

Using $\int_{-\infty}^{+\infty}\exp(-x^2/A)dx = \sqrt{\pi A}$, the y and s_0 integrations can be done immediately and, after applying the rotations (1), the following is obtained

$$\mathcal{L} = \frac{N_1N_2fB}{4\pi^2\sigma_s}\cos^2\frac{\phi}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\sigma_x^2\sigma_y}e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}}e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}dxds.$$

Similarly, the x integration may be done to yield

$$\mathcal{L} = \frac{N_1 N_2 f B}{4\pi^{\frac{3}{2}}\sigma_s} \cos\frac{\phi}{2} \int_{-\infty}^{+\infty} \frac{e^{-\frac{s^2 \sin^2\frac{\phi}{2}}{\sigma_x^2}} e^{-\frac{s^2\cos^2\frac{\phi}{2}}{\sigma_s^2}}}{\sigma_x\sigma_y\sqrt{1+(\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2}} ds. \tag{2}$$

If it is assumed that there is no hourglass effect, so any dependence of either of the transverse beam sizes on the longitudinal coordinate s may be neglected, a further integration of (2) (in s) gives

$$\mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y} \frac{1}{\sqrt{1+(\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2}} \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}. \tag{3}$$

From (3), one can recover the approximate expression for the luminosity of a bunched beam with a crossing angle. In particular, $\sigma_s \gg \sigma_x$ and so the first square root in (3) can be approximated to 1 and

$$\mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y} S, \tag{4}$$

where S is known as the luminosity reduction factor and is given by

$$S = \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}, \tag{5}$$

where $\tan\frac{\phi}{2} \simeq \frac{\phi}{2}$ for small ϕ is used.

1.1 Evaluation of Luminosity for the LHC

The evaluation of (3) for the LHC requires the following parameters:
$N_1 = N_2 = 1.1 \times 10^{11}$, the number of particles per bunch, with 2808 bunches per beam, $f = 11.2455$ kHz, $\phi = 300$ μrad, $\beta_x^* = \beta_y^* = \beta^* = 0.5$ m, $\sigma_x^* = \sigma_y^* = 15.9$ μm and $\sigma_s = 7.7$ cm, so

$$\mathcal{L} = 1.203 \times 10^{34} \times 0.809 \text{ cm}^{-2}\text{s}^{-1} = 9.73 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1} \tag{6}$$

where the first two numbers represent the maximum luminosity (without crossing angle) and the reduction factor, respectively.

1.2 Hourglass effect

In the above calculation, the so-called hourglass effect was not included, which is usually a good approximation. However, this may not always be the case (e.g. small β^*) and the reduction factor is calculated again numerically in the more general case below. Returning to (2), keeping $\sqrt{1+(\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2} = 1$, and putting

$$\sigma_z = \sigma_z^* \sqrt{1+\left(\frac{s}{\beta^*}\right)^2},$$

where $z = x, y$, σ_z^*, β^* are the values of the transverse beam sizes and beta function at the interaction point, (2) may be reduced to

$$\mathcal{L}_{HG} = \left(\frac{N_1 N_2 f B}{4\pi\sigma_x^*\sigma_y^*}\right)\frac{\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s}\int_{-\infty}^{+\infty}\frac{e^{-s^2A}}{1+(\frac{s}{\beta^*})^2}ds, \tag{7}$$

where

$$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x)^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} = \frac{\sigma_s^2\sin^2\frac{\phi}{2} + (\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\cos^2\frac{\phi}{2}}{(\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\sigma_s^2}.$$

The last integral has to be done numerically and gives

$$\mathcal{L}_{HG} = 1.203\times 10^{34}\times 0.806\ cm^{-2}s^{-1} = 9.69\times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}, \tag{8}$$

where, as before, the notation used is that the first term is the maximum luminosity and the second is the new reduction factor due to the combined effects of a crossing angle and the hourglass. So the inclusion of the hourglass effect ($\mathcal{L} \to \mathcal{L}_{HG}$) amounts to a reduction in the luminosity of 0.4 % and it may be concluded that, in the case of a bunch length of 7.7 cm and a β^* of 50 cm this effect may be neglected. However, the hourglass effect can become important and this is illustrated in figure (2) where the bunch length is taken as (10 cm) and the values of β^* considered are $5, 25$ and 50 cm.

2 Detailed analysis of the reduction factor and Luminous region

The density of the interaction vertices (where events are located) is a function of the beam sizes $(\sigma_x, \sigma_y, \sigma_s)$, the bunch length and the overall geometry. Therefore, a careful analysis of the luminous region or

$$\mathcal{L} = \int_{-s}^{+s}\mathcal{L}(s')ds'$$

is required. Using the program written for this purpose [2], several cases were considered and are listed below:

Eleven bunch lengths are considered ranging from 5.5 cm to 15.5 cm and increasing in steps of 1 cm at a time. At first, a zero crossing angle (figure (2) is considered for comparison. The results shown are with the vertical axis in units of 1.203×10^{34} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ and the horizontal one representing the distance s either side of the IP (given in cm). In the subsequent figures, the same bunch lengths are considered, but the crossing angle is 300 μrad (LHC) and then increased to 400 μrad and 500 μrad (figures (3), (4) and (5), respectively). All integrals were shown to converge (in the case of figure (2) to 1 and for the others lower) but only the region from 0 cm to 30 cm is shown as this is the one of interest.

3 Conclusions

The luminosity for the LHC was evaluated and, in particular, the luminous region lying 30 cm either side of the interaction point was looked at for a sample of crossing angles

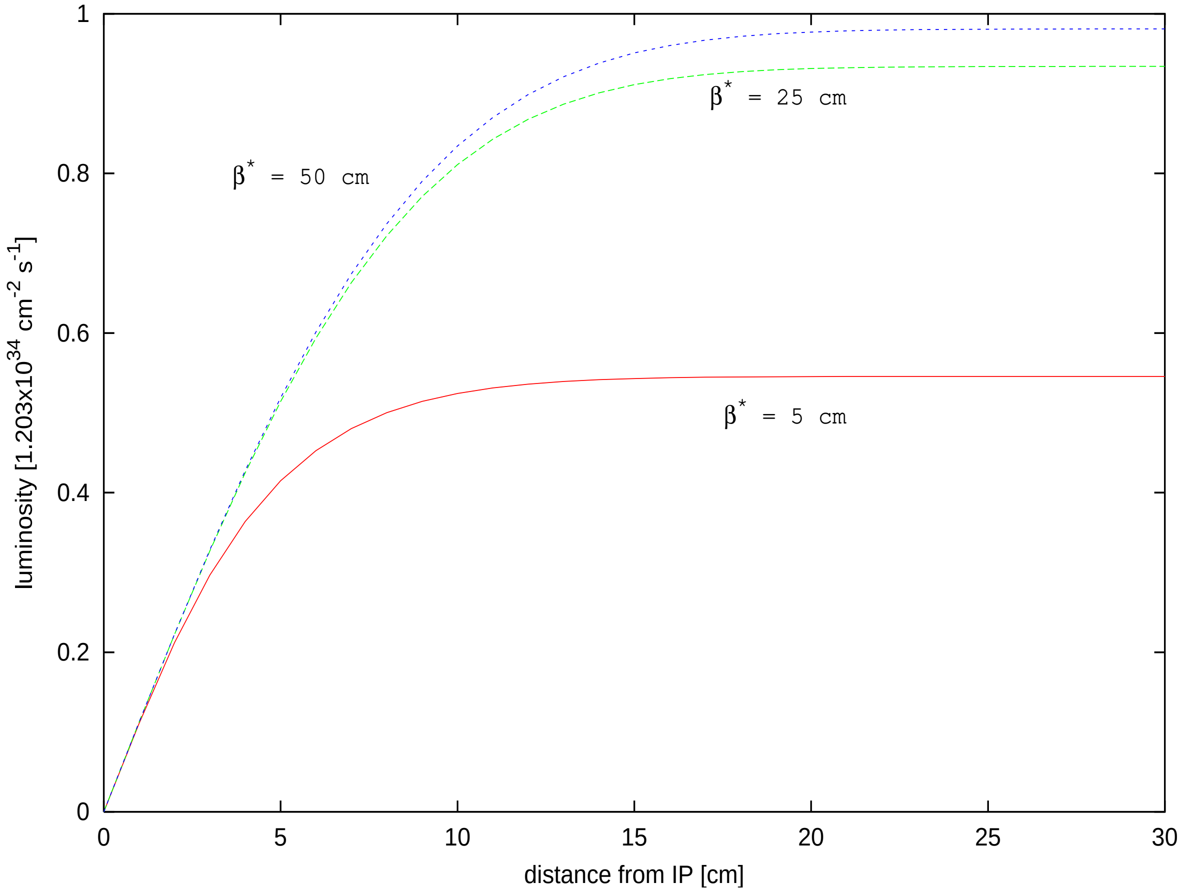

Figure 2: Hourglass effect for different β^* and a bunch length of 10 cm

and bunch lengths near the LHC parameters. It was also shown that, for the parameters considered, the hourglass effect may be neglected.

4 Acknowledgements

The author thanks W. Herr and M. Hayes for very useful comments and a careful reading of the manuscript.

References

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[2] B. Muratori, "Program 'lumi.f' " available at '/b/bmurator/public/lumi/lumi.f', (2002).

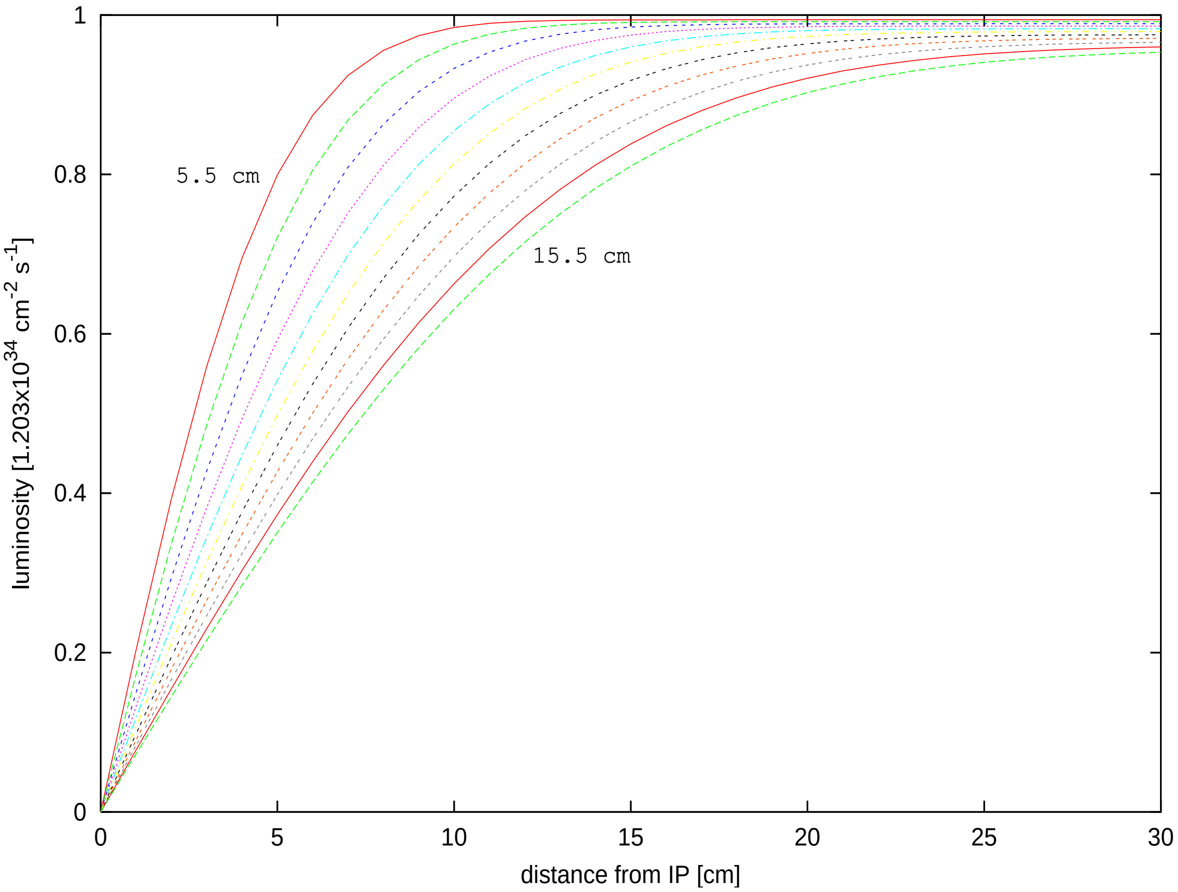

Figure 3: Luminous region for $\phi = 0.0\mu$rad

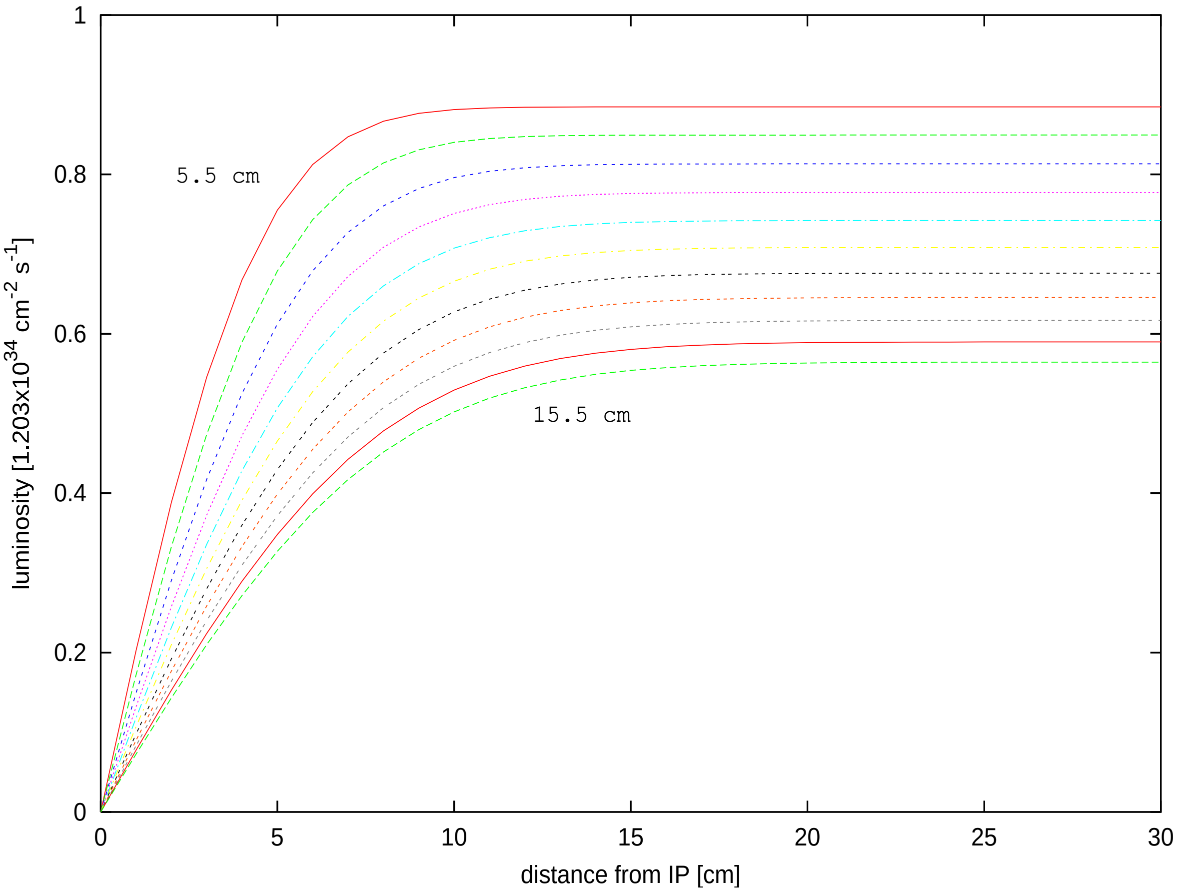

Figure 4: Luminous region for $\phi = 300\mu$rad

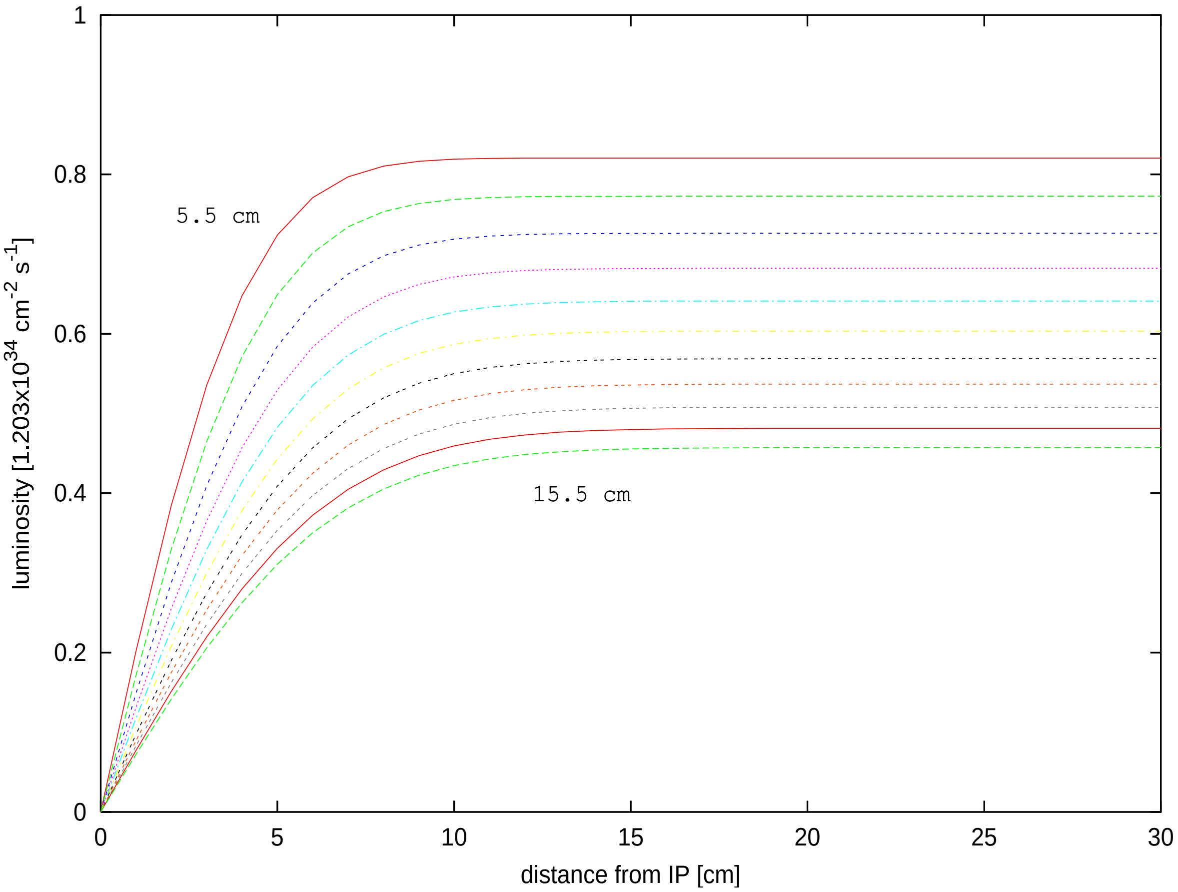

Figure 5: Luminous region for $\phi = 400\mu$rad

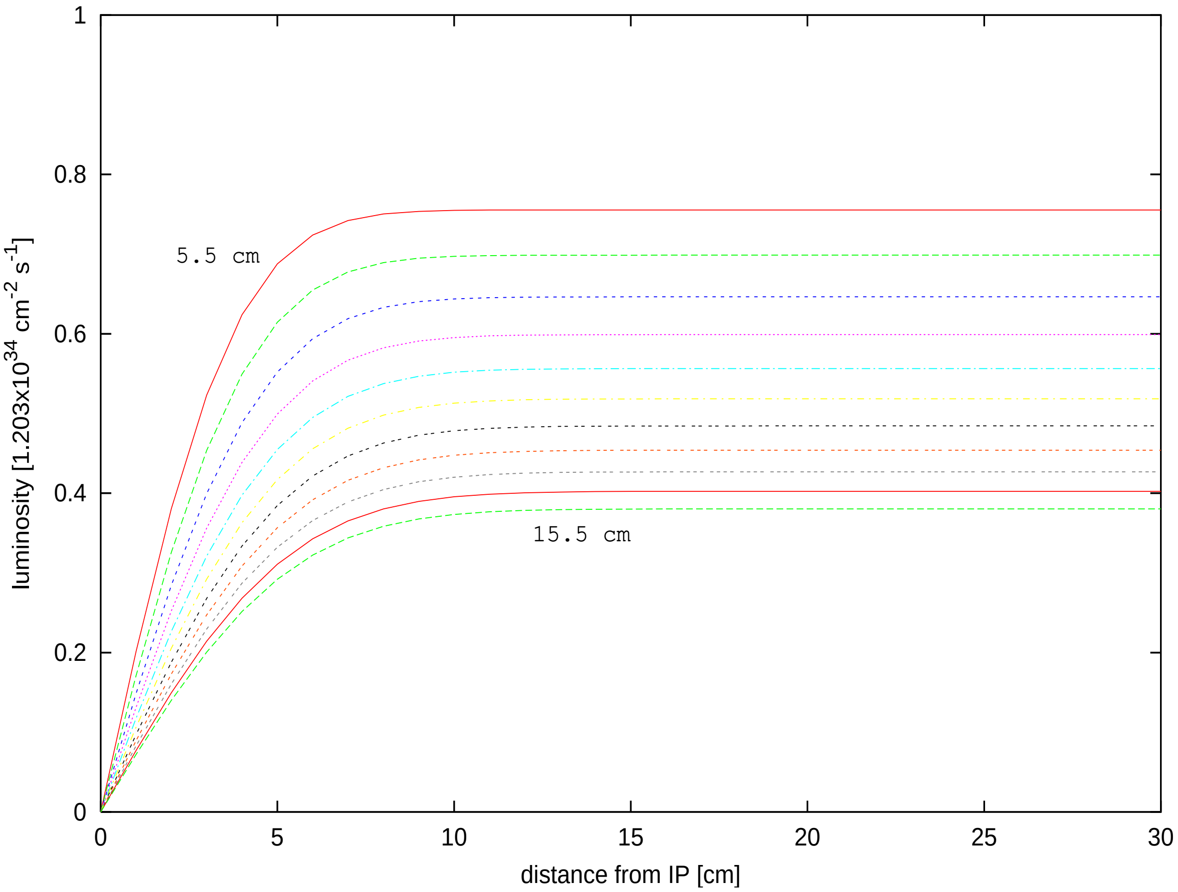

Figure 6: Luminous region for $\phi = 500\mu$rad