Luminosity Considerations for the LHC

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Introduction

- **Luminosity**
- **Luminous region**
- **Features**
- **Crossing angle**
- **Hourglass**
- **Calculations for the LHC**
- **Some results**
- **Computer program**
- **Conclusions**

Luminosity

- $$\frac{dR}{dt} = \mathcal{L} \times \sigma$$

- $$\mathcal{L} = 2N_1 N_2 f B \cos^2 \tfrac{\phi}{2} \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$
$$\rho_x^1(x)\rho_y^1(y)\rho_s^1(s - s_0)\rho_x^2(x)\rho_y^2(y)\rho_s^2(s + s_0)$$

- **In principle: should know all distributions**

- **In practice: only r.m.s. of ρ's relevant**

- **Choose Gaussian distributions for calculation**

- $$\rho_z^i(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\exp\left(-\frac{z^2}{2\sigma_z^2}\right) \qquad i = 1, 2, \quad z = x, y$$

- $$\rho_s^i(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}}\exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

- **Simplest case** $\rightarrow \mathcal{L} = \dfrac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y}$

- $\sigma_z \neq \sigma_z(s), \qquad z = x, y$

Luminous region

- **Where do events happen ?**

- **How big is the region where they happen ?**

- **Longitudinally** $(\pm s) \rightarrow \mathcal{L}(s) = \int_{-s}^{+s} \mathcal{L}(s')ds'$

- **ISR: coasting beam - crossing angle** $18\,^\circ$

$$\mathcal{L} = 2.0 \times 10^{31}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Luminous region with** $s = \pm\ 3$ cm

- **SPS: no crossing angle**

- $\sigma_x^* = \sigma_y^* = 60 - 100\ \mu\mathrm{m}, \quad \sigma_s = 30 - 40$ cm

- **Luminous region with** $s = \pm\ 30$ cm

Features

- **Particles/Bunch & # Bunches**

- **Transverse beam sizes**

- **Longitudinal beam sizes**

- **Geometry**

- **Crossing angle**

- **Hourglass**

Crossing angle

- $$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = x\sin\frac{\phi}{2} + s\cos\frac{\phi}{2}, \\ x_2 = -x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_2 = x\sin\frac{\phi}{2} - s\cos\frac{\phi}{2} \end{cases}$$
- **Crossing Angle** $\Rightarrow \mathcal{L} = \dfrac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y} S$
- S **is the reduction factor**
- $$S = \frac{1}{\sqrt{1 + (\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}$$
- **Small crossing angles &** $\sigma_s \gg \sigma_{x,y}$
- $$\Rightarrow S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

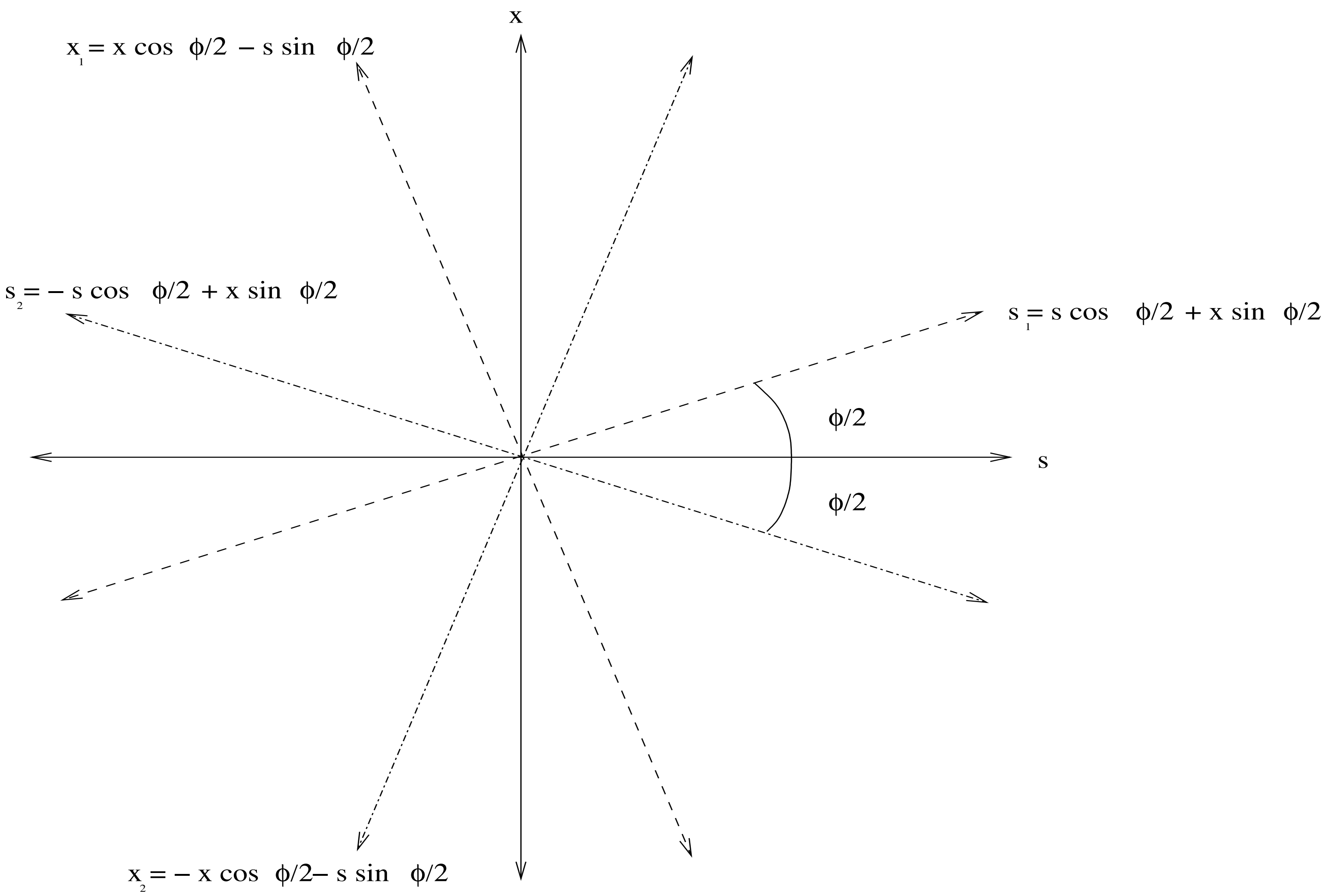

Figure 1: Geometry of crossing region

Hourglass

- $\sigma_z = \sigma_z^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2}$
- $\rightarrow$ **Integral too difficult**
- $\mathcal{L} = \left(\frac{N_1 N_2 f B}{4\pi \sigma_x^* \sigma_y^*}\right) \frac{\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} \frac{e^{-s^2 A}}{1 + (\frac{s}{\beta^*})^2} ds$
- $A = \frac{\sigma_s^2 \sin^2\frac{\phi}{2} + (\sigma_x^*)^2[1 + (\frac{s}{\beta^*})^2]\cos^2\frac{\phi}{2}}{(\sigma_x^*)^2[1 + (\frac{s}{\beta^*})^2]\sigma_s^2}$
- $\rightarrow$ **Numerical Integration**
- **program written & available**

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Calculations for the LHC

- $N_1 = N_2 = 1.1 \times 10^{11}$ **particles/bunch**

- 2808 **bunches/beam**

- $f = 11.2455$ kHz, $\phi = 300$ μrad **(total** $\times$ $\angle$**)**

- $\beta_x^* = \beta_y^* = 0.5$ m

- $\sigma_x^* = \sigma_y^* = 15.9$ μm, $\sigma_s = 7.7$ cm

- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.203 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

- **Crossing angle:**

$$\mathcal{L} = 0.973 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

- **Crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

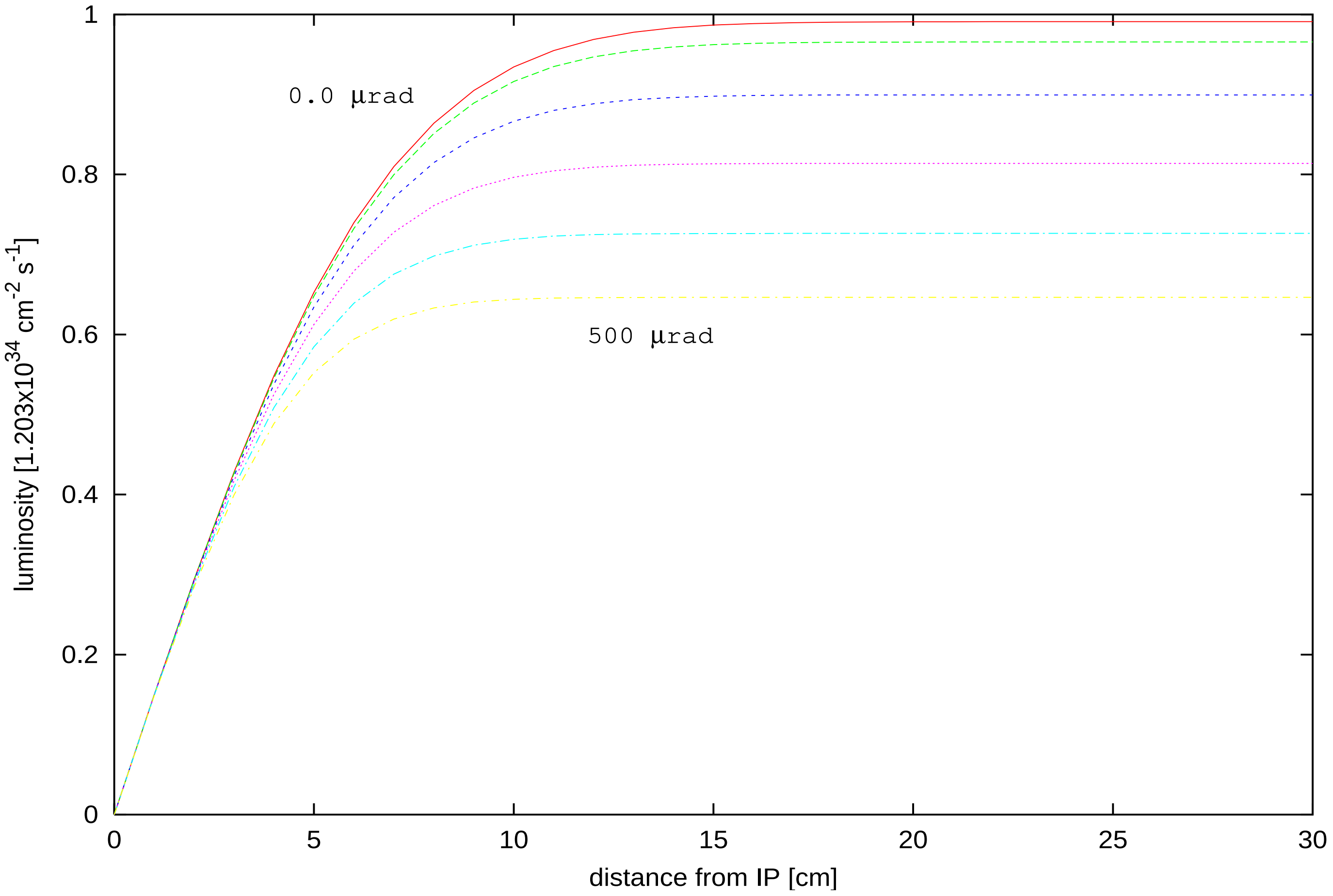

Figure 2: l. region for b.l. 7.5 cm, $\beta^* = 50$ cm and different $\times \angle$

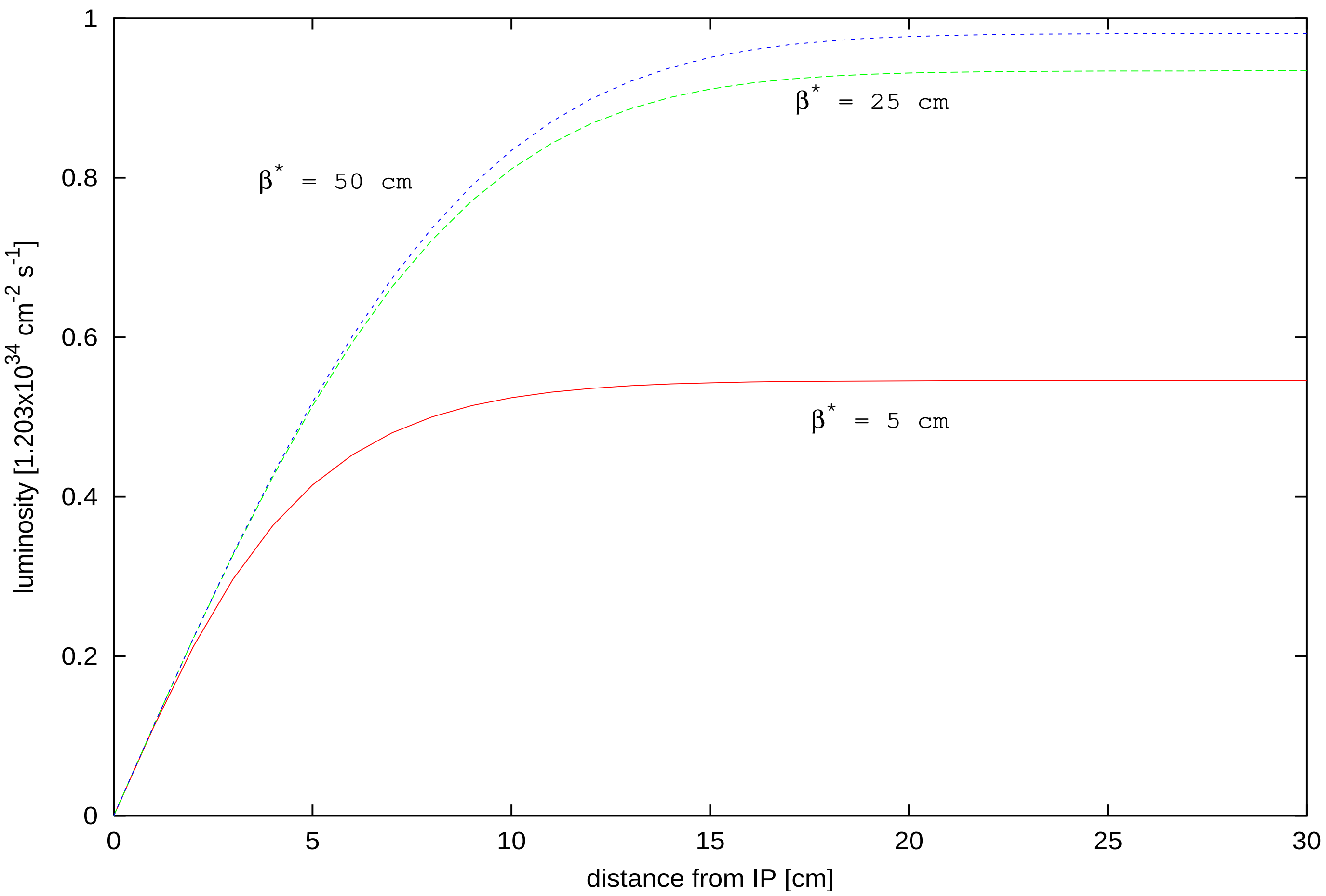

Figure 3: Hourglass effect for different β^*, b.l. $= 10$ cm $\phi = 0.0$ μrad

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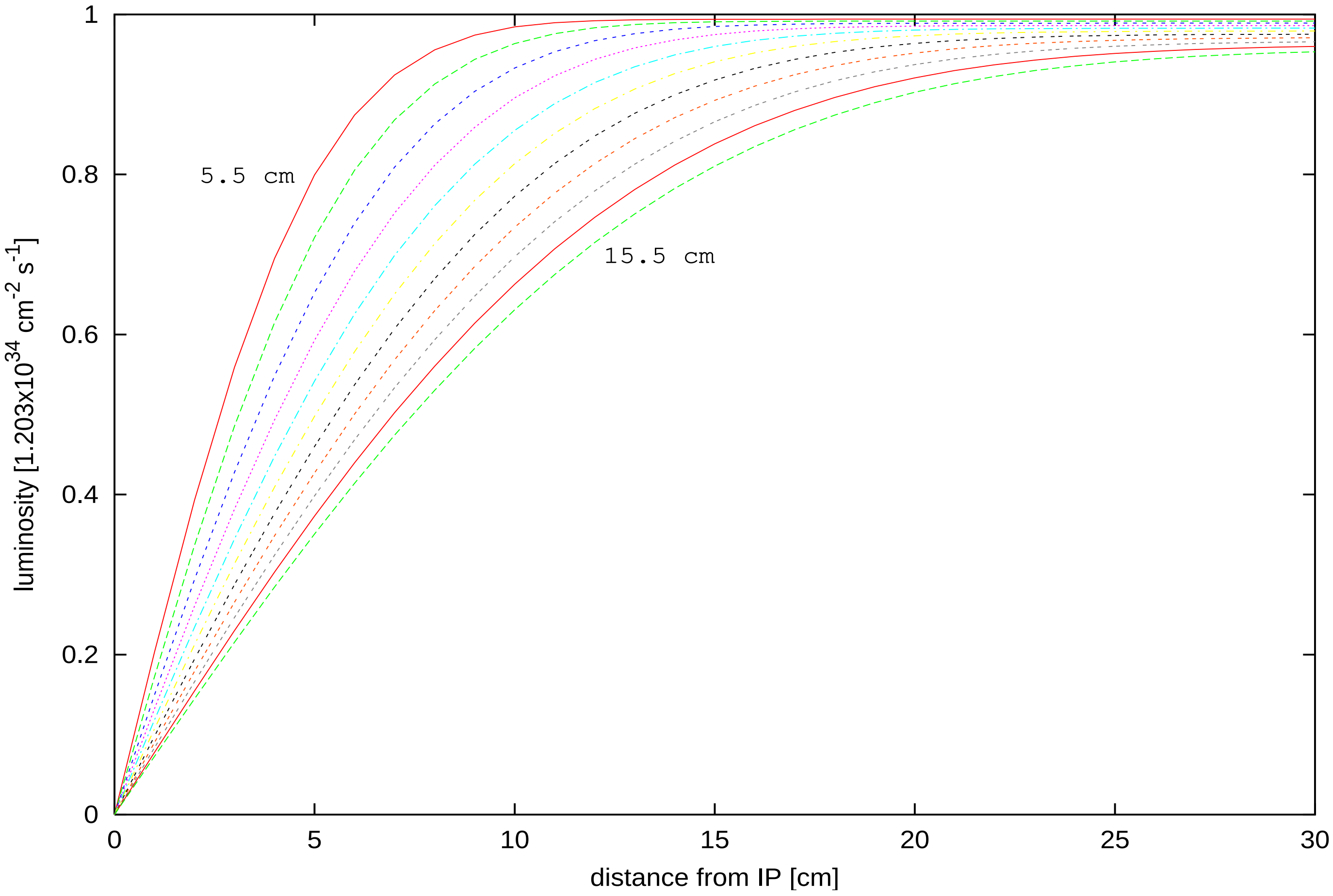

Figure 4: Luminous region for $\phi = 0.0\ \mu$rad, $\beta^* = 50$ cm

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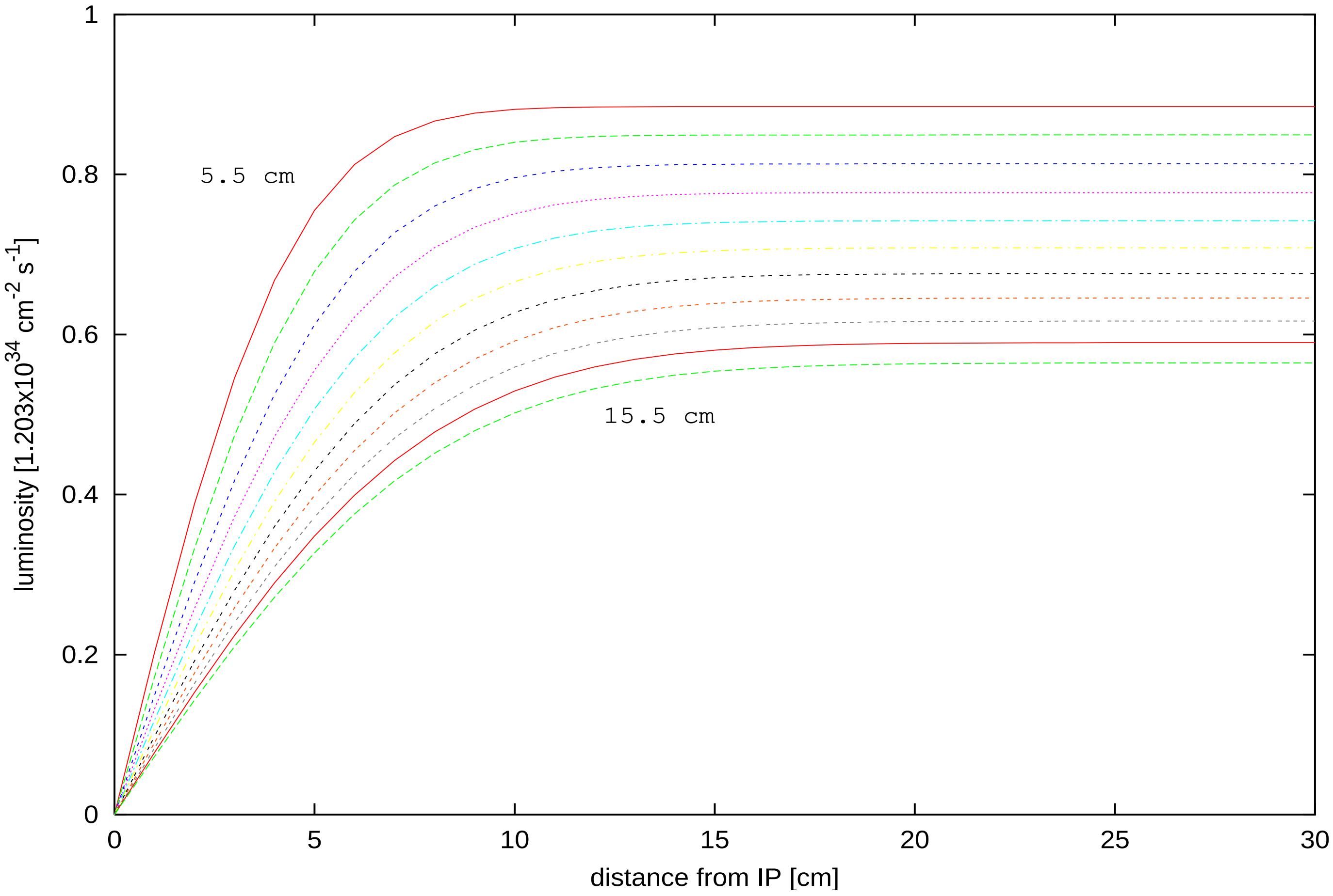

Figure 5: Luminous region for $\phi = 300\ \mu$rad, $\beta^* = 50$ cm

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Some results

- $\sigma_s = 7.5$ cm, $\beta^* = 50$ cm, $\phi = 300\ \mu$rad**:**

- 100% **lumi** $\rightarrow$ $s = \pm\ 12$ cm

- 95% **lumi** $\rightarrow$ $s = \pm\ 8$ cm

- 90% **lumi** $\rightarrow$ $s = \pm\ 7$ cm

- 85% **lumi** $\rightarrow$ $s = \pm\ 6$ cm

- 80% **lumi** $\rightarrow$ $s = \pm\ 5.5$ cm

Computer program

- **Available in public directory (lumi.f)**

  /afs/cern.ch/user/b/bmurator/public/lumi/

- **Different values of Bunch length** $(5.5 - 15.5 \text{ cm})$

- **Different values of** β^* $(5 - 55 \text{ cm})$

- **Possibility of different crossing angles**

- **Detailed results also available**

  /afs/cern.ch/user/b/bmurator/public/lumi/results

Conclusions

- **Angle of** $300\ \mu\text{rad} \Rightarrow 20\ \%$ **reduction of** $\mathcal{L}$
- **Higher reduction for larger angle**
- **Hourglass not important for** $\mathcal{L}$ **with current settings**
- **May become important for longer bunch length and/or lower** β^*
- **Luminous region calculated for several cases & available**